#### **Tevatron transverse instability study at low chromaticity.**

(A. Burov, P. M. Ivanov, C. Y. Tan)

# Main goal of this study

It is necessary to identify a driving mechanism for this instability, as well as to find possible solution for operation at small chromaticity.

#### **Introduction**

Operation of Tevatron at a high level positive chromaticity looks somewhat unusual and, probably, results in a degradation of machine performance. At the injection energy E=150 GeV the horizontal and vertical linear chromaticity equals to  $\mathbf{x}_{x,y} = P \frac{\partial \mathbf{n}}{\partial P} \cong +8.5 \text{ units}$ , but at the collision energy E=980 GeV it is increased up to  $\mathbf{x}_{x,y} \cong +26 \text{ units}$ . These large values of the chromaticity are needed to suppress a transverse instability of proton beam that can arise at some conditions.

#### **Head-Tail Instability Model**

Let us analyze at first, a possibility of the Single-Bunch Head-Tail Instability driven by the short-range wake fields in Tevatron. We will use a phenomenological approach in the study of this instability applying so called "air-bag beam model". One of the most important parameters used for description of the head-tail instability is the head-tail phase. In general case it is defined as a ratio of the betatron tune spread  $\Delta \boldsymbol{n}_b$  to the synchrotron frequency  $\boldsymbol{n}_s$ :

$$c = \frac{\Delta n_b}{n_s}$$

If the betatron tune spread is induced by non=zero linear chromaticity  $\mathbf{X} \neq 0$  that

$$\boldsymbol{c} = \frac{\boldsymbol{x} \cdot \Delta p / p}{\boldsymbol{n}_{s}} = \boldsymbol{w}_{\boldsymbol{x}} \cdot \boldsymbol{t}_{L} = \frac{2 \boldsymbol{p} \cdot f_{0} \cdot \boldsymbol{x} \cdot \boldsymbol{t}_{L}}{\boldsymbol{h}}$$

Here  $\mathbf{W}_{\mathbf{x}}$  is a chromatic betatron tune spread;  $\mathbf{t}_{L}$  is an effective proton bunch length;

$$h = a - 1/g^2$$
 is a slip factor;

In the framework of this model the transverse monopole mode m=0 is stable at positive chromaticity  $\mathbf{X}_{x,y} > 0$  and unstable if  $\mathbf{X}_{x,y} < 0$ . The higher order head-tail modes m>0 are unstable at  $\mathbf{X}_{x,y} > 0$  and stable at  $\mathbf{X}_{x,y} < 0$ .

In the range  $0 < c \le 1$  the growth rates of all higher head-tail modes m > 0 are proportional to the head-tail phasor.

If the coherent stability condition  $\Delta \boldsymbol{n}_b > 1/t_m$  is satisfied at  $\boldsymbol{c} \cong 1$  that all modes should be stable within  $0 < \boldsymbol{c} \leq 1$ . (model-independent).

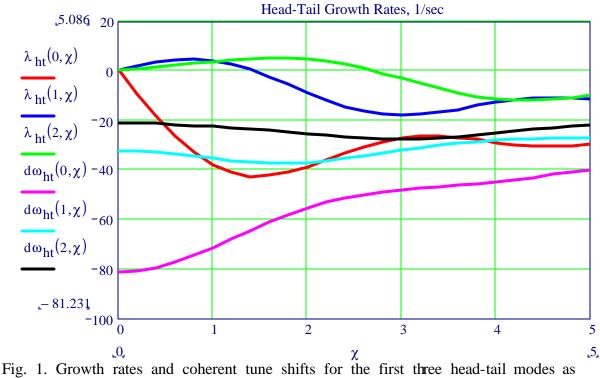


Fig. 1. Growth rates and coherent tune shifts for the first three head-tail modes as functions of the head-tail phase for the Tevatron at injection energy. Single coalesced bunch with  $N_{ppb} = 2.6 \cdot 10^{11}$  and resistive wall as a single source of impedance are assumed. Calculations are done in accordance with the "air-bag beam model" (A. W. Chao. Physics of Collective Beam Instabilities in High Energy Accel., Eq. 6.213, p. 350). The figure above shows that the coherent shifts are about 6 times smaller than the synchrotron frequency  $w_s = 540$  1/sec, thus, the accepted weak head-tail model is approved. Another significant issue is that the calculated growth rates are about of order of magnitude smaller than the synchrotron frequency spread  $Dw_s = 0.1w_s = 50.1$ /sec. Thus, the Landau damping have to stabilize all the modes. The conclusion is that the resistive wall impedance is at least 5 times smaller than the head-tail threshold at positive chromaticities. However, the total broad-band impedance of the Tevatron is calculated as approximately twice of the resistive wall one (Run II Handbook, Fig. 6.38).

At the Tevatron performance parameters (E=150 GeV):

$$f_0 = 47.7 \cdot 10^3 \; Hz$$
,  $\mathbf{S}_L = 3.0 \; n \sec$ ,  $\mathbf{t}_L = \sqrt{3} \cdot \mathbf{S}_L$  (Gaussian beam),  $\mathbf{h} = 0.0028$ , for  $\mathbf{X} = 1$  the head-tail phase equals  $\mathbf{c} = 0.54$ 

At low chromaticity, the most strong of higher modes is the dipole mode m=1, the growth rate of which reaches maximum value in vicinity of  $\boldsymbol{c}_1^* = 0.9$  that corresponds to  $\boldsymbol{x} \approx 1.7$ . For the quadrupole mode m=2  $\boldsymbol{c}_2^* = 1.8$  at  $\boldsymbol{x} \approx 3.3$ 

As the chromaticities are increased when  $c \ge 3 (x \approx 5.5)$  the dipole and quadrupole head-tail modes can be stable. Experimentally at the injection energy we didn't observe developing any transverse coherent instabilities until  $x \ge +6$ .

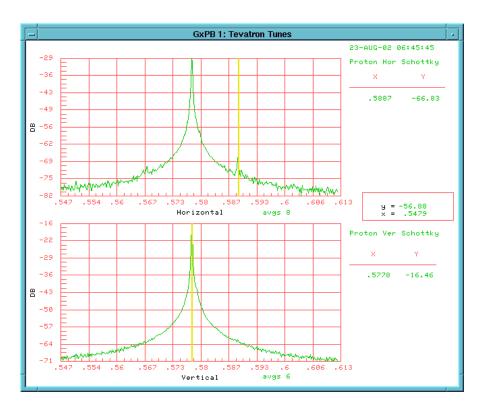
# **Experimental observations**

These observations were carried out at the Tevatron injection energy and standard lattice parameters. Chromaticities were set at injection as

$$\boldsymbol{X}_{x} \approx \boldsymbol{X}_{y} \approx +8$$

It was suggested that the transverse coherent instability could be initiated by decreasing the chromaticities after injection. The most interesting aspects of these studies consisted in observation in the Schottky monitor a growth of the coherent component in betatron spectrum. Also, it has been seen a longitudinally-correlated proton beam density loss after excitation of the coherent transverse instability. If there is a coherent excitation (transverse) of the proton bunch it will appear as a betatron frequency signal with an amplitude that is proportional to  $N_{ppb}$  (sharp peak), but not to  $\sqrt{N_{ppb}}$  as for the Schottky spectrum.

# Vertical coherent betatron instability of the uncoalecsed proton beam



$$N_{tot} = 2.5 \cdot 10^{11}$$
, Helix off

Betatron tunes:  $[n_x] = 0.5887$ ,  $[n_y] = 0.5778$ 

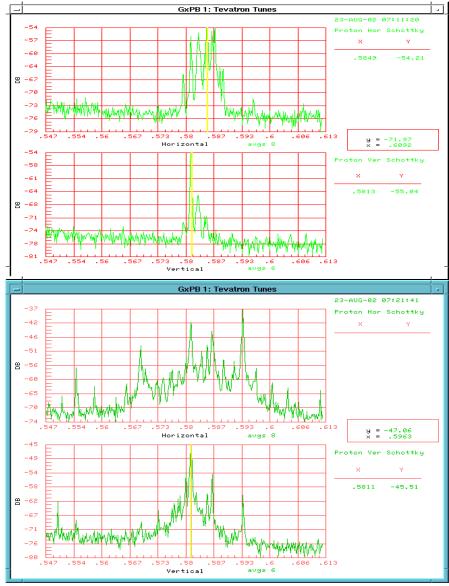
Cromaticities:  $\boldsymbol{x}_{x} = 4.7$ ,  $\boldsymbol{x}_{y} = 0.0$ 

85~% of the beam was lost at the chromaticities measurement (changing the revolution frequency)

## Coherent instabilities of a coalesced bunch

## Observation of the quadrupole mode

Before injection of the high intensity coalesced proton beam, an uncoalecsed beam was used for decreasing the betatron coupling and setting betatron tunes.



Betatron Spectrum for uncoalecsed and coalesced proton beams at the same Tevatron lattice parameters.

$$[\boldsymbol{n}_x] = 0.5849, \ [\boldsymbol{n}_y] = 0.5811, \ |\boldsymbol{n}_x - \boldsymbol{n}_y| = 0.0038, \ \boldsymbol{X}_x = +7.89, \ \boldsymbol{X}_y = +7.60$$

$$N_{ppb} = 2.6 \cdot 10^{11}, \ N_{tot} = 2.4 \cdot 10^{11}$$

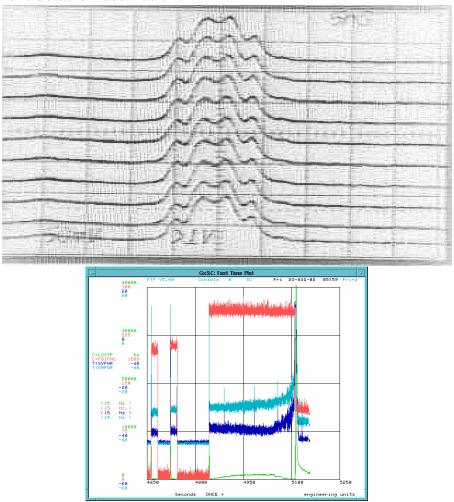
Spectrum of the coalesced bunch is much wider than that allowed by the chromatic tune spread  $Dn_x = x Dp / p \gg 0.006$ . Thus, the observed high remote peaks cannot be explained by the coherent tune shift. Note that actually the horizontal spectrum is wide, and the vertical only reflects it; this could point on the dispersion as an important factor in this phenomenon.

Transverse coherent betatron instabilities were initiated in both planes by decreasing the chromaticities down to  $\boldsymbol{X}_{\chi} \approx \boldsymbol{X}_{\chi} \leq +6$ . After the partial beam

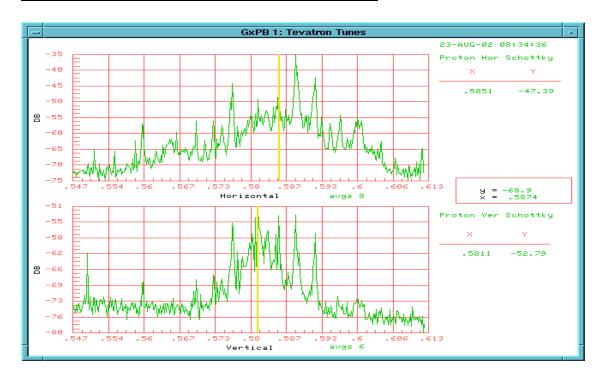
loss, we observed the stable beam distribution due to a self-stabilized development of this unstable mode.

$$N_{ppb} = 2.6 \cdot 10^{11} (init. beam) \Rightarrow N_{ppb} = 1.03 \cdot 10^{11} (remain. beam)$$

Shape of the remaining beam longitudinal density points qualitatively at excitation of the quadrupole coherent betatron mode m=2. The bunch lost the particles from its longitudinal density distribution, the synchrotron oscillation amplitudes of which are correlated with amplitude maximums of the coherent betatron oscillation.



# Observation of the coherent dipole mode m=1

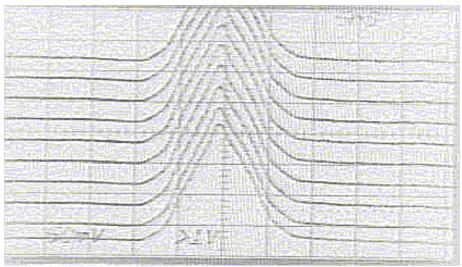


One coalesced bunch  $N_{ppb}$  =  $2.65 \cdot 10^{11}$  at the same lattice parameters.

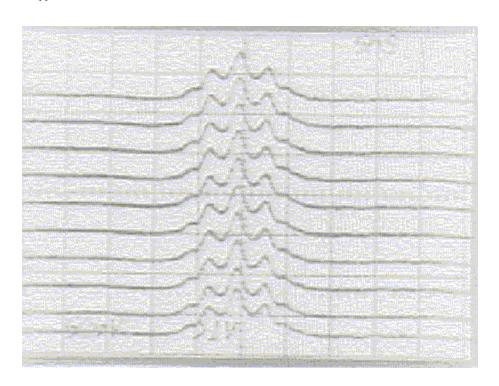
$$[\boldsymbol{n}_x] = 0.5851, \ [\boldsymbol{n}_y] = 0.5811, \ [\boldsymbol{n}_x - \boldsymbol{n}_y] = 0.0040, \ \boldsymbol{X}_x \approx +8, \ \boldsymbol{X}_y \approx +8$$

Vertical coherent instability was induced by decreasing vertical chromaticity down within the range  $+3 \ge x_y \ge +2$ . Estimated value of horizontal chromaticity should be within of  $x_y \approx 5-6$ .

$$N_{ppb} = 2.65 \cdot 10^{11} (init. beam)^{3} \Rightarrow N_{ppb} = 0.7 \cdot 10^{11} (remain. beam)$$



Original shape of the proton bunch longitudinal density with  $N_{ppb}$  =2.65·10<sup>11</sup>



Shape of the remaining beam longitudinal density points qualitatively at excitation of the coherent vertical dipole mode m=1.

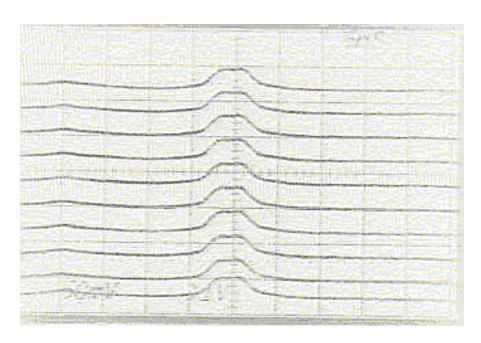
### Observation of the coherent betatron monopole mode m=0

One coalesced bunch  $N_{ppb} \approx 0.75 \cdot 10^{11}$  at the same lattice parameters.

Betatron tunes: 
$$[\boldsymbol{n}_x] = 0.5855$$
,  $[\boldsymbol{n}_y] = 0.5806$ ,  $[\boldsymbol{n}_x - \boldsymbol{n}_y] = 0.0049$ 

Vertical coherent betatron instability was induced during decreasing the vertical chromaticity down from the measured value of  $\mathbf{x}_y = +2.5$  to a negative range.  $\mathbf{x}_x \approx 0.3$ .

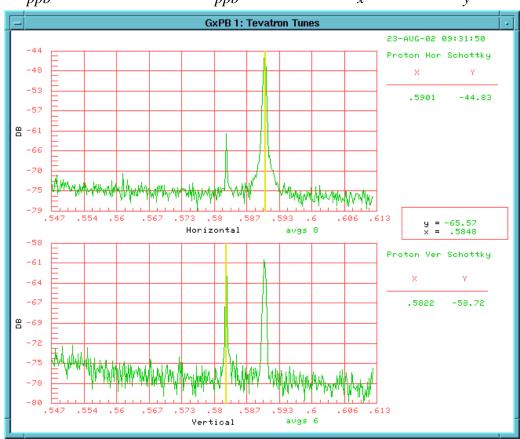
$$N_{ppb} = 0.75 \cdot 10^{11} (init. beam) \Rightarrow N_{ppb} = 0.34 \cdot 10^{11} (remain. beam)$$
 After the part of beam was lost we measured both chromaticities and detected that  $\mathbf{X}_x = +5.8$  and  $\mathbf{X}_y = -4.7$ 



Shape of the remaining beam longitudinal density points qualitatively at excitation of coherent monopole betatron mode m=0 at the negative vertical chromaticity.

Betatron spectrum of single coalesced bunch of low intensity at the negative horizontal chromaticity. Horizontal coherent instability.

$$N_{ppb} \approx 0.35 \cdot 10^{11} \implies N_{ppb} \approx 0.2 \cdot 10^{11}, \, \boldsymbol{x}_x = -1 \text{ and } \boldsymbol{x}_y = 0.$$



# Summary of the observations

Some EM fields are responsible for the excitation of the coherent betatron instabilities in both vertical and horizontal direction. The question is whether these fields are beam-driven (head-tail hypothesis, HTI) or beam-independent (single-particle effect caused by external fields the frequencies of which are in resonance with the betatron spectrum, SPE).

- 1. There is confirmation that the coherent betatron instability is a single bunch effect. The transverse coherent instability for multi-bunch mode of 36 coalesced bunches with  $N_{tot} \approx 71.5 \cdot 10^{11}$  or  $\langle N \rangle_{ppb} \approx 2 \cdot 10^{11}$  was excited by decreasing of the vertical chromaticity down to zero at  $\mathbf{X}_y \approx +0$  and  $\mathbf{X}_x \approx +7.5$ . Almost same behavior is observed for single coalesced bunch.
- 2. Threshold of the Instability is current-dependent.
- 3. Instability is chromaticity-dependent.
- 4. Experimentally observed three first coherent modes are qualitatively in agreement with prediction of the head-tail air-bag beam model.
- 5. Quantitatively the Tevatron resistive wall impedance is at least 5 times smaller than it is needed for the observed head-tail threshold at positive chromaticity. However, the total broad-band impedance of the Tevatron is calculated as approximately twice of the resistive wall one (Run II Handbook, Fig. 6.38).

Nevertheless, we cannot say that the head-tail phenomenon is entirely responsible for excitation of transverse coherent oscillations with a consequent partial beam lost. However, its role cannot be definitely excluded on a base of existing knowledge. On the other hand some aspects

of the instability observations strongly point at that the real picture of the coherent instability excitation looks more complicated as compared with the single head-tail phenomenon.

Although it is unclear which external EM fields might be concerned in the instability growth but the most likely, the Tevatron RF cavities can additionally affect on the transverse coherent beam dynamics by means of the higher order transverse modes or fundamental mode due to a presence of dispersion function. Moreover, the Tevatron lattice within the F0 straight is not optimal from this point of view- too large beta- and dispersion functions.

- 1. The betatron spectra are straying for any current. Even when we changed nothing, they changed time to time up to 10 dB in the power and 2\*10^(-3) of peak tune.
- 2. Spectra of the coalesced bunch are always wider than that allowed by the chromatic tune spread  $Dn_x = x Dp / p \gg 0.006$ . Thus, the observed high remote peaks cannot be explained by the coherent tune shift.
- 3. Horizontal spectrum is wider, and the vertical only reflects it; this could point at the dispersion as an important factor in this phenomenon.
- 4. Strong remote betatron peaks are not seen when the bunch population is 10% or so from the coalesced nominal (after 90% of beam lost or for uncoalesced bunches).
- 5. Multiple abrupt beam losses happened several times at very different  $N_{ppb}$  (from full  $2.6\cdot10^{11}$  to 10 times lower), sometimes when we did just nothing, sometimes during manipulation with RF frequency at chromaticity measurement, beam got coherent and (40-60%) lost.

# **Proposals**

1. In order to decrease possible influence of the head-tail mechanism or external EM fields on the instability current threshold it will be more optimal to operate at low positive chromaticities with introducing a cubic non-linearity by octupoles:

$$0 \le \boldsymbol{x}_{x,y} \le 1$$
 and  $\frac{\partial \boldsymbol{n}_{x,y}}{\partial (a_{x,y}^2)} < 0$ ,  $(\Delta \boldsymbol{n}_b)_{oct} \cong 0.002$ 

 $+2 \le x_{x,y} \le +6$  is the most dangerous range of chromaticities from the head-tail effect point of view.

Octupoles generate a betatron tune spread due to a non-linear tune dependence on the betatron amplitude that is more effective as compared with the chromatic tune spread produced by sextupoles;

2. It is necessary to study the possible influence of the RF cavities on the threshold of coherent instability (decreasing beta- and dispersion functions if it is possible).